RiXtrema Global Market Liquidity (GML) Model

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[1] Section IIIc is applicable to the both the RiXtrema GML and Asset Allocation risk models.

Appendix A: GML Fixed Income Factor Breakdown

I. Factor Models and Risk Forecasting

Traditionally, a multivariate normal distribution has been assumed in a Modern Portfolio Theory framework for the returns of financial securities. This is a direct consequence of measuring risk as a variance of the return, since in absence of the assumption of normality variance would be a non-unique descriptor of the portfolio risk profile. In other words, without the Gaussian distribution (or, to be more precise, an elliptical distribution) assumption, there could exist a number of portfolios with the same variance, but with drastically different loss potential and therefore drastically differing risk profiles. When a multivariate normal distribution is assumed for the returns of financial securities, the key practical issue that emerges is how to estimate the parameters of such distribution, specifically the interrelationships between the assets. For a two asset portfolio (composed of assets A and B), the portfolio variance formula is:

$$\sigma_P^2 = w_A^2 * \sigma_A^2 + w_B^2 * \sigma_B^2 + 2 * w_A * w_B * \rho_{A,B} * \sigma_A * \sigma_B \qquad (1)$$

Where:

σ_A - Standard deviation of asset A (B)

w_A - Weight of asset A (B) in the portfolio

$\rho_{A,B}$ - Correlation of asset A and asset B

In matrix notation, this equation could be expressed as:

$$\sigma_P^2 = w' * D * w \qquad (1A)$$

Where:

w - Row vector of portfolio weights with the length equal to the number of assets

D - Covariance matrix of all assets

It is easy to see from formula (1) that it requires estimation of 3 parameters, two standard deviations and one correlation coefficient. However, this number grows very quickly as assets are added to the model. With only 1,000 assets we will require 500,500 parameters to be estimated for our multivariate distribution. The return data used for such estimation is available at best for a decade or two, thus making the number of observations much smaller than the number of parameters in this hypothetical model. Needless to say, the statistical properties of such coefficients would be highly suspect.

The main purpose of a factor model is to improve the quality of the statistical estimates of risk. This is done by using the compact description of the securities' return via the factor model. One type of a factor model that uses time series of factor returns as an input is written as follows:

$$R_i = r_f + \beta_{i,1} * F_1 + \beta_{i,2} * F_2 + ... + \beta_N * F_N + \varepsilon_i \qquad (2)$$

R_i - Vector of returns for security i across time

r_f - Nominal risk free rate

$\beta_{i,1}$ - Scalar exposure of security i to factor number 1

F_j - Vector of returns across time for factor j

ε_i - Vector of error terms for security i

It can be shown that equation (2) can allow us to describe the risk of any combination of assets for which we know the exposures and residual terms. The equation would look like this:

$$\sigma_P^2 = w^{'} * B * C * (w^{'} * B)^{'} + w^{'} * G * w \qquad (3)$$

Where:

B - Matrix of exposures (loadings) of each asset on each factor in the model

C - Variance/Covariance matrix for all factors

G - Diagonal matrix of residual term variances

II. Problems With Traditional Risk Models

a. Shocks and Discontinuities

Modern Portfolio Theory (MPT) started with the work of Harry Markowitz and is based on a number of fairly elaborate assumptions regarding the financial markets. MPT's dominance of the portfolio management and reporting resembles the ubiquitous presence of Newtonian mechanics in the building of automobiles. Not surprisingly, MPT comes under scrutiny every time a supposed once-in-a-lifetime event occurs in the financial markets. It is the unrealistic assumption set underlying the MPT that usually bears the brunt of each failure in the eyes of the investing public; this is particularly true for a bell curve distribution axiom. Most critics have disparaged the bell curve for its lack of fat tails. However, upon close examination we come to the paradoxical conclusion that, despite the basic correctness of the assessment first made by Benoit Mandelbrot (1963) regarding the presence of fat tails in the financial markets, the inappropriateness of the bell curve is really one of the least important problems with traditional risk models. To get at the roots of the problem, we must go back to Barr Rosenberg (1973), the father of all present day risk modeling. In his significantly titled "Prediction of Systemic and Specific Risk in Common Stocks" paper, he started with these momentous words:
"Ex Ante predictions of the riskiness of common stocks – or, in more general terms, predictions of probability of returns can be based on fundamental (accounting) data for the firm and also on the previous history of stock prices." The discussion of questions that this little paragraph appears to have so impressively and easily settled could have filled volumes, and the empirical results appeared solid enough for widespread acceptance. Why were the results so good, given what we now know about the performance of the CAPM based risk models? We believe that the key reason for acceptance of Rosenberg's paradigm and its MPT foundations was the particular sample that he used for testing, the years from 1954 to 1970. This happened to be one of the most tranquil periods in the history of stocks, the standard deviation of the daily returns of S&P 500 over this period was meager - around .66% (annualized volatility of 10.46%), while the same daily standard deviation since that time (1970-2010) was much higher at 1.08% (annualized volatility of 17.1%). The standard deviation of S&P 500 from 1928 to 1954 was even higher at 1.47% (annualized volatility of 23.2%). Another and perhaps a more illuminating way

of looking at this issue is to count the number of days when S&P 500 was down more than 3.5 %[2]. For the sixteen year period between 1954 and 1970 the number of days with a return below 3.5% was only 3 (!). For the prior sixteen years, namely between 1938 and 1954, the number of such days was 24, eight times higher. Meanwhile, the last sixteen years between 1994 and 2010 have produced 35 such days. Rosenberg was not the only one who developed his ideas during a period of stable economic growth and absence of major systemic risks. As Peter Bernstein (2007), himself an ardent proponent of efficient markets, notes, all of the key pillars of Modern Portfolio Theory were put in place between 1951 and 1973. Surely, in such a calm environment as 1950's and 1960's the financial markets may appear continuous and recent historical returns are really all you need to forecast risk. Thus, much of the groundwork of the MPT risk modeling was laid during the period of unusually low risks and this fact has to be kept in mind by anyone wanting to understand the roots of the problem.

Some modifications were later introduced, but the key conclusions regarding modeling of risk held up. They can be summarized thus – to forecast risk we only need to know past volatility of factors and average correlations between them along with the characteristics of the assets and their loadings on factors. In other words, the implicit assumption of the old paradigm is that volatility of future returns is a mirror of the volatility of past returns; that there are no discontinuities or sudden jumps in volatilities. This ultimately means that low volatility of actual returns and correlations calibrated mostly over the tranquil periods will ALWAYS produce low volatility estimates, as it happened, for example in 2007 and even 2008. The problem we are pointing out is known and resurfaces after every crisis, as it did in January of 2009 when Basel Committee said: "Most risk management models, including stress tests, use historical statistical relationships to assess risk. They assume that risk is driven by a known and constant statistical process… Given a long period of stability, backward-looking historical information indicated benign conditions so that these models did not pick up the possibility of severe shocks or the buildup of vulnerabilities within the system". Regarding this quote it might be said that 'better late than never', except that even this stark admission seems to be all too frequently ignored when firms try to improve risk management processes using the same old foundation. In order to address this problem RiXtrema developed an asset class behavioral risk model that is geared toward analyzing and forecasting the endogenous risk, see Danielsson and Shin (2003) and Satchkov (2010), of the economy. The GML

[2] This cutoff is close to three times the long run daily standard deviation (1928-2010) of 1.169%, so it provides a good metric to qualify extreme events.

(Global Market Liquidity) model is built to address other deficiencies in the MPT paradigm, which are potentially even more dangerous than missing the buildup of endogenous risk. For more detail on the RiXtrema Asset Allocation model email mail@rixtrema.com. The rest of this paper will deal with other major problems of the MPT based models and will show how RiXtrema GML holdings based model is solving them to properly estimate risk of multi-asset portfolios.

b. Tail Correlation/Diversification Assumptions

In practice, Barr Rosenberg's suggestion about "…history of past stock prices" serving as the foundation for the calculation of risk forecasts became a rule that covariance matrices are calculated over some recent history, such as two, three or five years. There are a number of grave problems with such approach. The most obvious one, as discussed above, is that such a model cannot possibly (by its very construction) capture discontinuities in the market and will frequently show low risk just before a shock. However, a deeper and possibly an even more serious problem is the mis-estimation of the correlations that takes place with such an approach. This point is worth exploring in some detail.

One of the key conclusions of the MPT is that in a portfolio with more than a handful of assets, the correlation between those assets is likely to be a key factor in determining the portfolio's risk level. In other words, diversification can transform the combination of high risk assets into a low risk portfolio if correlations are low enough. In addition, Barr Rosenberg's statement about accounting data and past history of the returns being enough to forecast volatility has suggested to practitioners an elegant way of modeling these correlations and constructing portfolios in a seemingly rigorous and objective manner. In essence, it led to the introduction of a new strong assumption, which is much more consequential than the beleaguered bell curve:

- Future volatility and correlations are a function of statistical estimates of volatility and correlations calculated over some recent period[3]

The estimation of correlation matrices over the recent periods produces a number of problems beyond the fact that the model may show low risk right before the crash[4]. Diversification benefits that serve as the

[3] Recent history was used because, it was reasoned, the markets are not static and change continuously. Exponential decay is also frequently used to overweight recent observations making the model more responsive.

foundation for identifying the best risk-return tradeoffs are calculated using those very matrices. When those supposed risk reducing diversification benefits vanish exactly as they are needed most, we frequently hear sighs about a 'rise in correlations'.

As Rick Bookstaber (2007) put it:

> "Investors are not as dumbfounded when volatility skyrockets as when correlations go awry. This may be because investors depend on correlations to control their risk and to allow them to extend further out in their investment exposures. And nothing hurts more than to think that you are well hedged and then to discover you are not hedged at all."

It is important to understand that the term "rise in correlations" can be misleading. We do not observe correlations; all we are really saying is that our model (MPT) breaks down in extreme environments, i.e., it doesn't perform exactly when the risk manager needs it most.

So firm is the grasp of MPT-type diversification on the public's mind that almost exactly ten years after the spectacular meltdown of LTCM, Donald Kohn the Vice Chairman of the Federal Reserve emphasized the concept of "International Decoupling" at the International Research Forum on Monetary Policy. "International Decoupling" is an essentially MPT-inspired idea that gained great traction in the media and among analysts in the months leading up to the September 2008 crash. It suggested that international markets were shielded from possible housing related problems in the U.S. because world economies interacted with each other directly more than in the past and interacted somewhat less with the U.S. economy. Relatively low average correlations that prevailed between the US and emerging equity markets between 2003 and 2007 served as the justification for this view. Subsequent events have exposed this idea as absolutely unrealistic when the markets are under stress, especially under the conditions of global mobility of capital. As we know, MPT concepts embedded in the issuance and valuation of CDS and MBS did not fare much better.

For a thorough discussion of the problems with downside correlations, see "The Myth of Diversification" by Chua et. al. (2009). In that article, which is based on extensive and rigorous empirical tests, authors state the following: "Correlations, as typically measured over the full sample of returns, often belie an

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[4] After all, missing of a crisis rarely puts the survival of the firm in question. Future is inherently uncertain and most investors will miss most individual financial crises by definition.

asset's diversification properties in market environments when diversification is most needed[5]." This problem is not an accidental feature of one or two crises, but rather a permanent property of the financial economy in stress mode. RiXtrema models explicitly take account of the fact that tail correlations are significantly and persistently different from tranquil periods. However, simple methods of shocking correlation matrices or limiting the sample to 'extreme' periods are not acceptable solutions (for more on that see section IIIC).

III. RiXtrema Global Market Liquidity (GML) Risk Model[6] Technical Description

RiXtrema model is built in a number of distinct steps with each having its own goal. First, the factors are defined and loading of each security onto the factor is calibrated using time series multiple regression (described in section IIIa). Secondly, we use RiXtrema's unique method to gather and statistically examine those residuals that were observed in periods when a given local equity market was under stress. This allows the user to estimate how much 'unexpected' illiquidity risk a given security might be hiding (Section IIIb describes this step). We then estimate a joint multivariate distribution for the factors. To do this, we separate the distribution into the marginal components for each factor and into a copula that covers the interrelationships between the factors. The way the model incorporates the copula is one of the crucial innovations in the RiXtrema GML model. We start out with the standard Gaussian copula for the factor relationships, but change the weighting structure to give more importance to the periods of global fear, which exhibit a different structure and frequently evoke complaints of 'rise in correlations'. After overweighting the left tail, we implement RiXtrema's patent-pending method to find the correlation matrix most consistent with the observed left tail data. This procedure allows us to capture significantly different joint relationships in the tail, essentially solving the problem that we outlined in much detail in section II (section IIIc describes the 'tail copula' procedure itself). Marginal distributions are modeled as the last step by taking standard exponential averages (3 year history with a .96 decay constant). The marginal part of the estimation resembles traditional models (described in section IIId). Finally, residual risks are estimated by taking periods of local market stress and isolating the remainder of the variability that cannot be easily explained by the most important principal components. This procedure allows us to

[5] Despite the excellent quality of research and clear problem formulation, we do not view the non-parametric optimization proposed as a solution in that particular paper as practically helpful.

[6] Multiple patents are pending for risk forecasting methods mentioned in this document.

estimate the truly idiosyncratic risk without ignoring systemic influences (described in section IIIe) that appear only in crisis. Note, that this is fundamentally different from simply creating a hybrid model by using PCA on all the residuals, since residuals during extreme periods exhibit structure that is very different from the rest. Exhibit 1 summarizes the novel features of the GML model.

Exhibit 1 – Features of the RiXtrema GML Model

Feature of the Model	**Different from MPT based models**	**Importance**	**Section**
Loadings Estimation	NO	Allows to minimize the number of parameters and to decompose risks by factor.	IIIa
Crisis Liquidity	YES	Allows the user to understand the hidden liquidity effects bottom up i.e. specific to the local market and to the security in question.	IIIb
Tail Copula	YES	Allows to properly estimate available diversification benefits in the tail and to avoid being blindsided by the 'rise in correlations'.	IIIc
Marginal Distributions	NO	Estimating individual risk of each factor. To forecast crisis in a given asset class use RiXtrema's behaviorally based Asset Allocation model	IIId
Idiosyncratic Risk	YES	Estimated after accounting for 'Crisis Liquidity' during periods of local market stress and representing truly idiosyncratic risk.	IIIe

a. Factor Definitions and Loadings Estimation

At this stage, the goal is to provide factor descriptions using indices that are familiar to practitioners in order to analyze the risk exposures. The loadings estimation is done in a stepwise manner to avoid problems with multicollinearity and to prioritize the factors. The first step of the regression includes only local equity market for each security to get the primary beta of each security. The second step includes the global equity index, as well as macroeconomic variables oil and gold. Given the excessive money creation and clear potential for competitive currency debasement, gold exposure becomes an important dimension of risk. The third step includes two momentum indices, liquidity and exposure to multiple industries. The fourth step is a unique feature of the RiXtrema GML model. Most models start out with the assumption of liquidity and then include one separate liquidity factor, usually some variation of the volume-to-float ratio. RiXtrema models explicitly incorporate the idea that all risks eventually become liquidity risks. GML does include a standard liquidity factor, but goes beyond that by analyzing the extreme periods for each local index and performing statistical analysis to find commonalities in the stock residuals that were not observed during normal times. These common tail risk factors affect various equities to a different degree. So, we perform Principal Component Analysis (PCA) of the residuals in those specific states of the financial world to measure the effect of illiquidity on a given stock (more detail in section IIIb).

Exhibit 2 – Factor Definitions in the GML Model

Factor Name	Factor Definition	Regression Step
Local Equity Index	Size weighted return index of all (or the top 1000 by size, if the number is greater than that) securities in the particular local market	First
Oil	Oil return index	Second
Gold	Gold Return index	Second
Global Index	Size weighted global return index of top 3,000 global companies by size	Second
Size	Difference in the return of the top decile of companies globally by market cap minus the return of the bottom decile	Third
Growth/Value	Difference between the return of the growth and	Third

	value indices where growth/value is defined by price-to-earnings, price-to-book and dividend yield ratios.	
Perma-Liquidity	Difference in the return between the most liquid (top decile of volume/float) and least liquid (bottom decile of volume/float) segments of the world equity market	Third
Industries	Each security is exposed to 10 (out of 72). The ten industries are chosen by ranking all loadings in the order of highest to lowest statistical significance (statistical measure) and choosing the top 10	Third
Tail Liquidity	PCA is performed on the residuals in those periods when the local index had biggest absolute moves up or down. The resulting eigenvector is the vector of loadings of each security to these factors	Fourth
Yield Curve	Each yield curve is represented by the following 9 points on the curve: 6 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 20 years, 30 years. Each fixed income instrument is exposed to the appropriate yield curve. Riskostat has global coverage and includes capability to easily add new curves.	Not Applicable
Spreads	There are a number of spread categories. For example, Corporate bond spreads are bucketed into sector/rating category; Muni/MBS/CMBS bonds into rating categories.	Not Applicable

b. Tail Liquidity Principal Component Analysis (PCA) for Equities

Traditional risk models treat liquidity as a relatively permanent feature of the market that is separate from other components of risk. RiXtrema models differ from the traditional view in two important aspects.

First of all, liquidity risk in a normal period (i.e. the market impact type) is not the same as liquidity risk in the tail period; the latter one is triggered by a completely different set of mechanisms. Secondly, liquidity risk is not separate from other types of risk; rather it is the psychological and structural consequence of other risks jumping to the extreme. “Every risk eventually is a liquidity risk” it has been said and our statistical machinery is geared toward identifying and measuring this illiquidity. The first liquidity factor in our model called Perma-Liquidity is calculated as the excess return of most liquid companies over the least liquid, where liquidity is measured by the volume-to-float ratio. In essence, the definition of liquidity that is implied from this first liquidity variable is roughly “the degree of price discontinuity experienced on average when selling a large quantity of a given asset”. However, if we stopped there, we would be missing the most important risk variable of all – tail liquidity. Typical factor models are calibrated over long time series where tranquil periods will predominate and will tilt the beta estimation toward minimizing the overall error. However, this could mean that important systematic influences that are present only in the tail are completely ignored. In our testing, the structure of the residuals becomes drastically different when we go from looking at overall averages to focusing only on the tails. Exhibit 3 below shows a number of principal components that are required to explain 50% or 60% of variance of the residual in ‘normal’ versus ‘extreme’ periods. The table clearly shows the difference in market structure and the fallacy of pooling together the residuals from normal and extreme periods under the rubric ‘idiosyncratic risk’. For example in the S&P 500 example, 30 principal components are required to explain at least 50% of the residual variance in normal periods indicating that the residuals are close to being truly independent. In extreme periods, 50% of the residual variance generated from the same universe of securities can be explained with as few as six principal components. The difference in the number of principal components from one country index to another is due to the size of the universe in the sample i.e. this sample contains significantly more companies from the US large cap and Japan Nikkei than from the rest of the indices on the list. Note, even though there is a large difference in the size of the universe across the indices (and, therefore, in the number of PCs required to explain 50% of the residual variance) and the difference in the equity markets themselves (e.g. China and India versus the more developed markets), the number of orthogonal vectors required in extreme periods is remarkably similar from market to market. This supports the hypothesis that market microstructure is significantly different in extreme periods and also explains why factor models fail asset and risk managers in times of crisis.

Exhibit 3 – Number of principal components required to capture at least 50/60% of the residual variance in the periods of local equity crises

		60 Month Average Residual					30 Most Extreme Periods				
Country	**# equities**	**2007**	**2008**	**2009**	**2010**	**2011**	**2007**	**2008**	**2009**	**2010**	**2011**
united states	**2022**	**74**	**77**	**79**	**84**	**81**	**8**	**7**	**7**	**7**	**7**
china	**863**	**40**	**42**	**40**	**43**	**37**	**1**	**1**	**1**	**1**	**1**
japan	**784**	**64**	**65**	**69**	**72**	**71**	**5**	**6**	**6**	**6**	**6**
united kingdom	**384**	**39**	**40**	**41**	**42**	**43**	**7**	**7**	**8**	**8**	**7**
canada	**385**	**33**	**35**	**37**	**38**	**40**	**8**	**7**	**7**	**7**	**7**
hong kong	**290**	**27**	**28**	**30**	**31**	**31**	**3**	**3**	**2**	**2**	**2**
india	**265**	**32**	**33**	**34**	**35**	**36**	**5**	**5**	**4**	**4**	**4**
germany	**187**	**25**	**24**	**24**	**19**	**21**	**5**	**4**	**4**	**4**	**4**
australia	**208**	**23**	**26**	**27**	**27**	**30**	**5**	**6**	**7**	**7**	**6**
france	**228**	**23**	**26**	**28**	**27**	**29**	**6**	**6**	**7**	**6**	**7**
taiwan	**224**	**20**	**19**	**20**	**21**	**22**	**2**	**1**	**1**	**1**	**1**
korea, republic of	**173**	**22**	**20**	**20**	**20**	**19**	**1**	**1**	**1**	**1**	**1**
brazil	**114**	**14**	**14**	**15**	**15**	**16**	**2**	**1**	**1**	**1**	**1**

The definition of liquidity that is implicit in looking at the tails of the distribution in this way is "the degree of price discontinuity experienced in extreme environments when selling any quantity of a given asset". It is into this liquidity risk that other risks eventually turn. Two securities with the same volume-to-float ratios could exhibit very different degree of illiquidity in the tail based on this definition. So, an innocuous relative bet against the benchmark could actually hide a vicious surprise, just as an apparently neutral strategy could hide major directionality if this important aspect of risk is ignored.

Specifically, the following procedure is performed in order to capture these tail specific liquidity factors. After calibration of the factor model we will calculate residuals only in those periods when the local market index experienced significant volatility[7]. We then perform Principal Component Analysis on these extreme period residuals to find the commonalities observed in the Exhibit 3 above.

$$\varepsilon_{xi} = R_i - (\beta_{i,1} * F_1 + \beta_{i,2} * F_2 + ... + \beta_N * F_N + \beta_{i,LQ,PC1} * F_{PC1} ... + \beta_{i,LQ,PC5} * F_{PC5})$$

$\beta_{i,LQ,PCk}$ - is the eigenvector corresponding to extreme PC k (out of 5 PCs and corresponding to the asset i).

F_{PC1} through F_{PC5} are values ('returns') of the PCs for extreme periods.

ε_{xi} - Residual calculated only in only in the extreme periods

The eigenvectors of the PCs then serve as loadings of each security on this factor and the eigenvalue is the variance due to that PC. In order to understand the conceptual importance of this procedure in the GML model, it is useful to think about the type of problem that we are solving with this modeling device (versus the problem that is solved with the tail copula). The tail copula is addressing the global (both in terms of geography and asset types) shift in the correlation structure that invariably occurs in the tail, while 'extreme liquidity' addresses a specific liquidity differences that manifest themselves when a particular local market (e.g. UK equities) is in crisis. So, the two features serve important and highly complementary practical purposes to help users avoid being blindsided by the changes in the various aspects of the market structure during crises.

c. Tail-Implied Copula: The Simplistic and the Realistic

[7] We use both up and down moves to increase the effective sample, but due to a certain degree of skewness that is observed empirically, left tail observations usually predominate.

One of the unique aspects of the GML model is that marginal distributions and the copula are estimated separately. The marginal distributions use time-varying exponential averages, but the copula uses longer time series and uses tail observations combined with Bayesian analysis to imply the most realistic matrix that is consistent with what we actually observe in the tails of the distribution. The conceptual purpose of the tail-implied copula is as follows. Since factor interrelationships are significantly and persistently different than would be implied by the normal period correlations, traditional models severely mis-estimate the diversification benefits that are available to the user; for detail see Chua et al (2009) and Satchkov (2010B). So, a 'tail copula' is capturing how standard factors relate differently to each other in the tails of the distribution. In essence this is the global part of the 'rise in correlations' phenomena that we hear so often about. To put it another way, tail observations contain important information which, based on extensive experience of the practitioners and empirical considerations, is at odds with the output of traditional models.

1. Traditional Methods of Addressing "Rise in Correlations" and Their Fallacies

A number of risk modeling vendors are claiming that they have gotten a grip on the "rise in correlation" problem, which is arguable the most dangerous phenomena for a portfolio manager. The methods used for addressing this phenomena are:

- Conditioning a Sample Using Tail Events: Two-Way Conditioning
- Conditioning a Sample Using Tail Events: Left Tail Conditioning
- Shocking Correlation Matrices Subjectively
- Power Law Distributions

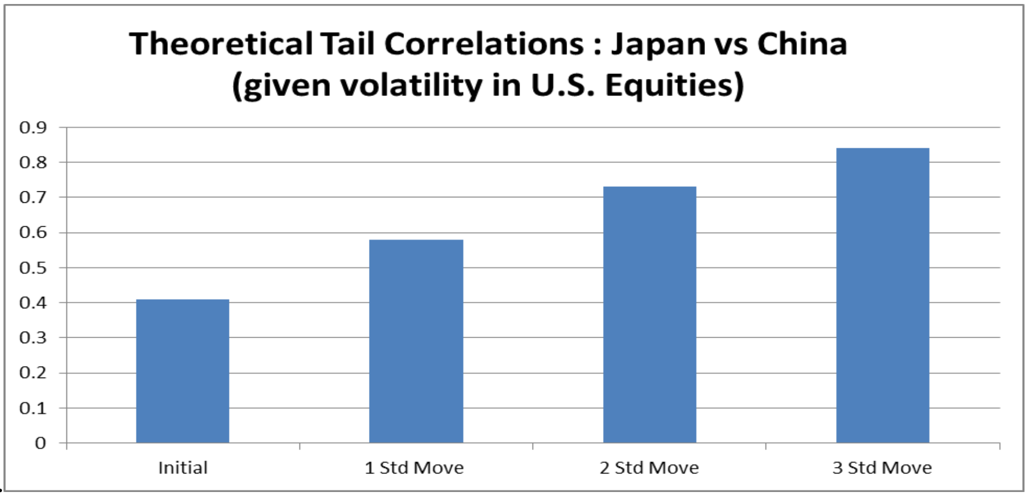
In this section, we will discuss each of these methods to address the "rise in correlations" problem, reasons why those methods are inadequare, and danger for application in the risk management practice.

Conditioning a Sample Using Tail Events: Two-Way Conditioning

When a sample is conditioned on a period being 'extreme' an indirect dependence of the sample on one of the model variables is introduced. Let's say we use the level of VIX as a definition of the 'extremeness' of the period. A more 'extreme' period under this method would be the one when VIX is high. It is obvious that during that period the US equities and by extension equity markets worldwide have experienced significant moves both to the upside and to the downside, though the downside moves would be more frequent in such a subsample. Consequentially, 'extreme' periods are those when one of the factors in a model (and any multi-asset class risk model will include a US equities-related set of factors) has moved a great deal and this makes any correlation obtained from this sample incomparable to the correlation that is the parameter of the whole distribution. However, it is routinely compared and even used in place of the correlation parameter to make the model 'tail aware'.

This conditioning problem is discussed in some detail in Ang and Chen (2002). Modeling tail correlation effects by selecting only tail periods presents a statistical fallacy. This fallacy presents itself under different guises depending on the type of conditioning. The first type of conditioning is to look at both tails. One example would be to look at periods when US equities (or some other factor) moved more than X standard deviations to the right or to the left i.e. two-way conditioning. In this case, even if the world was magically governed by a bell curve distributed statistical machine, the correlations measured in the subsample would automatically be higher simply based on the properties of the multivariate Gaussian distribution. To see that this is so, consider the following experiment. Start with a three factor model that includes US, Japan and China equities. The starting correlation between Japan and China is around .4. Now let us simulate this three variable distribution getting 10,000 scenarios. Note, that these scenarios are artificially constructed and could not by construction contain any real world effects that could be characterized as 'rise in correlations'. Now, let us select only those events where US Equity markets had larger moves (to approximate the tail periods) up or down. We can calculate the correlation between Japan and China equities under different cutoffs for the US Equity factor. When we only select those periods where US equity moved 1 standard deviation in either direction, the correlation between Japan and China becomes almost .6. For the two standard deviation moves in the US equities it is above .7 and it is approaching .84 under the three standard deviation move in US equities. Exhibit 4 below shows these results graphically.

Exhibit 4 – Statistical fallacy of naïvely conditioning the sample on 'tail events' based on two-way cutoff



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This simple experiment proves that 'rise in correlations' can be observed as a consequence of selecting data points based on some criteria that is related to the factor in a model and need not be caused by any fat tail effects. That is not to say that tail effects confounding risk practitioners are not real, but that they must be disentangled from the sample conditioning. To disentangle true shifts in correlations, RiXtrema uses a patent-pending unconditioning procedure to find the correlation matrix most likely to generate observed tail events.

Conditioning a Sample Using Tail Events: Left Tail Conditioning

As we saw above, a 'correlation rise' is a property of the joint normal distribution when sample is selected using two-way conditioning. A reasonable question that could be asked is the following: "What if we conditioned using only left tail observations?". The answer to that question can be seen in Exhibits 5 and 6.

Exhibit 5 – Correlations between US and India equity indices using one-way cutoff

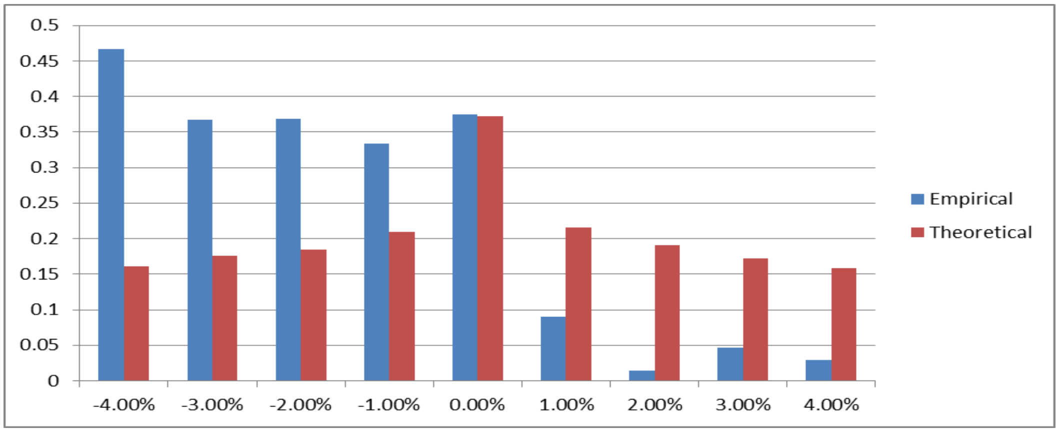
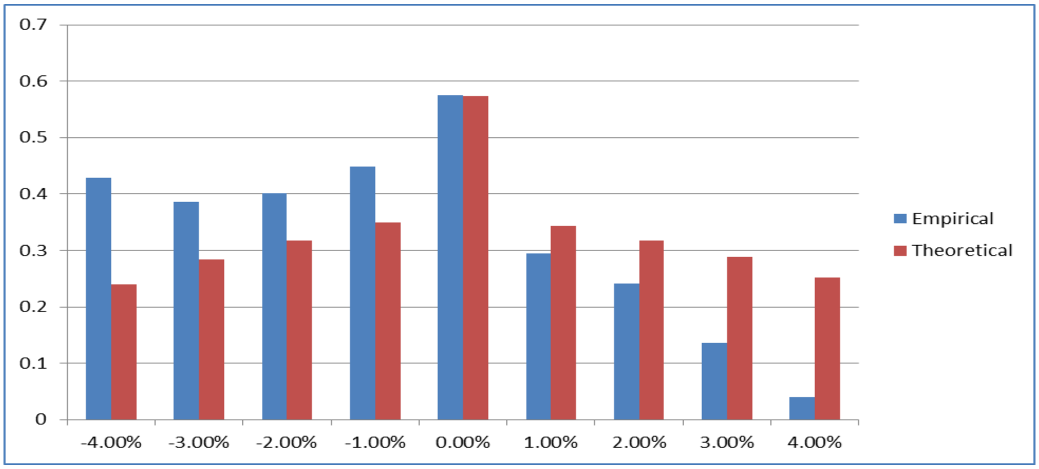


Exhibit 5 shows two sets of correlations between the US and India equity indices (1996-2012 using weekly returns). The Theoretical series is obtained by first simulating a bivariate US-India distribution that is initially estimated using the most common industry method – the decay time weighted method (lambda of .98). The simulated results are then filtered (conditioned) by using various subsamples based on the moves in the US index. So, a -3% on the x-axis indicates that only observations coinciding with US index losing more than 3% in any given week were collected. A 2% cutoff means that observations when US returned less than 2% were gathered. Thus, the Theoretical series is analogous to the series in Exhibit 4, except that a one-way cutoff is used. It cannot possibly contain any real-world effects that we know as 'fat tails' or 'rise in correlations', because it shows us what the world would be like if it were magically governed by the multivariate Gaussian distribution. Note, that the character of the tail correlations has changed, now instead of increasing, they are actually falling when the cutoff is moved away from the center of the distribution. This is an important point to keep in mind for any researcher that looks to make sense of various statements made about tail correlations. Anyone using exponentially decay weighted covariance matrices really depends on the world to look the way the Theoretical series do. Any significant departures from the Theoretical series behavior would make the results of risk analysis, stress testing and optimization that are based on Modern Portfolio Theory tools ill-suited for risk management.

However, the really striking observation pertains to the Empirical series obtained by applying the same cutoffs to the historic observations. Note that the Empiric series deviates significantly from the behavior prescribed by the Gaussian distribution, and does so in a distinctly asymmetric manner. The left tail correlations are significantly higher than prescribed by the normal distribution, while the right tail ones are significantly lower. This asymmetry is what is causing the investment managers to frequently obtain the worst of both worlds when assuming the Theoretic series (via the various Modern Portfolio Theory based models), while working in a world that exhibits characteristics of the Empiric series. Exhibit 6 contains a similar comparison of Theoretic and Empiric one-way conditioned correlation series for US and Brazil equity indices. See also Longin and Solnik (2000) for a thorough pre-2008 discussion of this issue with similar results. The overall pattern is similar, though not without some distinct features. Exhibit 4, 5 & 6 clearly show that a manner in which conditioning is conducted plays a crucial role in making sense of and taking advantage of the shifts in correlations that occur in the tails. Exhibits 5 and 6 in particular play in important role in the construction of all RiXtrema risk models, as they provide a motivation for the tail-implied copula structure.

Exhibit 6 – Correlations between US and Brazil equity indices using one-way cutoff



Shocking Correlation Matrices Subjectively

Another simple solution to the problem of tail correlations has historically been a stress test of the correlation matrix. In this method a whole matrix is scaled upward linearly to reflect something that is commonly thought to occur during tail events. But this method has no base in empiric data. In a number of detailed studies, for example, Chua et al (2009) it is quite clear that correlations shift in a non-linear fashion; some rise, some drop, some actually stay quite flat. Consequently, linear arbitrary shocks to correlation matrices produce no meaningful benefit in dealing with the problem

Power Law Distributions

The third solution that is sometimes tried is a power law copula. It is anything but simple, nonetheless it fails for a simple reason. Multidimensional problems require lots of data. A Gaussian model requires estimation of $\frac{N*(N+1)}{2}$ parameters. For a multidimensional model, the data that we have available already pushes the statistical robustness to the limit even for this basic type of model. A fatter tail distribution introduces a whole new layer of parameters that require estimation and is simply impossible to estimate with any degree of robustness until we have 10,000 plus years of financial data series.

2. RiXtrema Approach to "Rise in Correlations" - Tail-Implied Copula

Normal copula is described as follows, for more detail see Cherubini et al (2004). Denote $X = (X_1, \ldots, X_m)'$ a random vector of factors with zero means, its correlation matrix describes normal copula, while standard deviations of factors represent marginal distributions. To calculate the correlation matrix we first divide the factor returns sample with respect to the values of a key factor; the latter is taken to be the US local index, which is denoted X_1: as follows: calculate σ_1, the standard deviation of the key factor, and form "below" and "above" samples by including to X^B all the elements with $X_1 < -t\sigma_1$, and placing the rest to X^A. Here $t = -1.5$ is a threshold. Denote n^B, n^A number of samples in X^B, X^A. Next, calculate the conditional decay weighted covariance matrices by

$$c_{ij}^B = \frac{1-\lambda}{1-\lambda^{n^B}} \sum_{k=1}^{n^B} X_{ik}^B X_{jk}^B, \quad c_{ij}^A = \frac{1-\lambda}{1-\lambda^{n^A}} \sum_{k=1}^{n^A} X_{ik}^A X_{jk}^A, \quad i,j = 1,\ldots,m,$$

with decay $\lambda = .98$. Using the two covariance matrices C^B, C^A[8], we calculate a combined correlation matrix by applying the RiXtrema 'unconditioning' procedure (note that this is a form of Bayesian analysis):

[8] Zero mean assumption is applied for the following reason. Clearly, means of the conditional distributions corresponding to covariance matrices C^B, C^A are not zero. However, if we used empirical means in the calculation of covariance, we would have to account for that in our calculation of the implied distribution. This would introduce a whole new element (shift in means) into our risk forecasting. Setting mean to zero focuses tail-

$$C = C^B + \frac{1}{c_{11}^A} C_1^A C_1^{A\prime}, \tag{4}$$

where C_1^A denotes the first column of C^A[9].

The motivation behind the equation (4) is as follows. There exists a substantial, if not conclusive, body of evidence that left tail environments are entirely different from others and that diversification benefits generally forecast by average correlation risk models simply disappear in crises. Exhibit 5 and 6 are just some of the examples illustrating this phenomenon. Ideally, we would like to employ a multivariate distribution that would capture these joint tail effects through the fat tail structure, for example a power law distribution. However, due to reasons outlined in the Power Law Distributions section above, it is simply not possible with any reasonable degree of robustness. The standard Multivariate Gaussian Distribution calculated using equal weighted or time decay weighted approaches does not allow for the distinct left tail characteristics of the correlation regime observed in practice. The only way for a risk practitioner to proceed is to correct for this deficiency of the Gaussian. The best approach to this problem, in our view, is analogous to the way that traders use implied volatility to correct the Black-Scholes formula; "a wrong number to put into a wrong formula to get the correct price"[10]. The Copula part of the distribution (i.e. correlations) is analogous to the implied volatility in this type of reasoning. Nobody really saw a correlation rise any more than they saw a number of different volatilities for the same asset; the phrase 'rise in correlations' simply hides the fact that the model significantly deviates from reality, as does Black-Scholes when used with a single volatility estimate. Implied volatility is a number that matches the model to the real world. Tail-implied correlations do the same for the multivariate Gaussian. In our case, the 'real world' part of that statement is not the market price as it is for options traders, but a risk forecast or, more precisely, the diversification that is actually experienced during crisis events. The proxy for this diversification is the observed tail correlation matrix in crisis periods, i.e. the first term on the right hand side of formula (4). However, as we have seen above in the section Conditioning a Sample Using Tail Events, simply using (conditional) tail correlations in place of unconditional correlations in the

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implied algorithm on the key question we want to ask, namely, are common factor movements (including covariance and mean shift effects) larger than individual ones (variances)? Common factor movements that are larger (in absolute sense) than movements in individual variability imply correlation shifts and that is our goal.

[9] As described in the Marginal Distributions section and elsewhere in the document, our goal in the GML model is not change the estimation of variance, but rather of correlations which will dictate risk decompositions and stress tests. So, this formula is used to get the correlation part of the structure, but the variances are obtained by standard decay weighted methods.

[10] This quote has been attributed to Bruno Dupire among others.

model relies on a statistical fallacy, because conditioning on tail events is always related to the factors in the risk model. To avoid this problem, RiXtrema uses Bayesian inference to uncondition the problem to find the unknown joint Gaussian distribution that would be consistent with the observed left tail. This is why we called RiXtrema's copula 'tail-implied'; it is implied by actual tail observations.

Let us compare it briefly to the logic of the traditional MPT approach. Traditional models simply settle for the average (exponentially decay weighted or otherwise) covariance calculation to find this unknown distribution and, as a consequence, it ends up being inconsistent with the left tail observations, a fact that practitioners painfully rediscover time and again when crises occur. As a consequence of this misconception, traditional models frequently suggest diversification that works on the upside (when you need it the least) and fails on the downside (when you need it the most). RiXtrema models are looking to forecast the diversification that is implied by the actual tail events. Our goal is to find the original unknown distribution that is consistent with tail events and not simply extract it from observed average behavior aggregated across different regimes.

Consider again formula (4). The covariance matrix C^B is a covariance matrix of the distribution of model factors given that the driving factor (US Equities) moved more than 1.5 standard deviations to the left. The second term on the right of formula (4) arises out of the Bayesian adjustment to uncondition left tail observations for subsequent use in risk forecasting.

Diversification is not a myth, but it is routinely mis-estimated by a wide margin and practically every big failure of the financial firm in recent memory resulted not from missing a single crisis, but from the improper estimation of tail relationships. RiXtrema's method produces correlation matrices consistent with what is actually observed in the tail without falling into the pitfalls of traditional methods of dealing with tail correlations, such as linear correlation shocks, simple conditioning on factors related to the model and use of copulas with higher moments.

3. RiXtrema Tail-Aware Stress Testing

In general, RiXtrema advocates for a much more prominent role for stress testing and reverse stress testing and nowhere do the departures from exponentially decay weighted dogma affect a risk management workflow, than in these two areas[11].

Exhibit 7 contains some examples of the dramatic improvement imparted by the use of the tail-implied copula algorithm to the forecasting ability of a stress testing algorithm. The foundation for the mathematics behind factor stress testing was laid down by Kupiec (1998) and it can be found in a conditional multivariate Gaussian distribution. A shock to one of the factors is specified by the user and conditional distribution is used to calculate the conditional mean for the rest of the factors and consequentially for the assets exposed to those factors via the loading matrix. Mechanics of the evaluation of the accuracy of the stress testing algorithm have also been originated by Kupiec (1998). The essence of testing is to compare predictions from the algorithm to the actual realizations. In the specific case of stress testing, the following steps are taken:

- Extreme events of interest are isolated.
- A model (in this case we have two models: tail-implied and time decay weighted) is calculated on the eve of the extreme event without look ahead.
- A shock is specified that coincides with the actual events that subsequently transpired. Note that this is the only element of using future information. It is present here, because the goal of the stress testing algorithm is not the forecast of the likelihood of the crisis, but the correct forecast of the impact that the crisis will have on various portfolios i.e. contagion effects.
- Each model (in this case, we have the exponentially time decay weighted model and the RiXtrema's tail-implied model) is then compared to the actual return that has transpired subsequently and RMSE (root mean squared error) is calculated to assess the degree of accuracy.

The first test presented is a shock to Diversified Financials index made in a pre-Lehman collapse world in the beginning of September. The goal of our test is to see how various global portfolios (most of which are not holding the Diversified Financial sector directly) are going to respond to such a shock. Five hundred random portfolios are used and forecast from two models are compared with the actual return

[11] Optimization is the third area of major impact, but the discussion is beyond the scope of this paper.

observed in a subsequent period. Tail-implied model shows a RMSE (Root Mean Squared Error) of .007, almost one-fifth that of the MPT (Modern Portfolio Theory exponentially decay weighted model). A similar shock in 1998 (the LTCM crisis) produced a RMSE of .016 for the tail-implied model and .021 for the MPT model. More recently, a shock to Italy equity index in July of 2011 produced a RMSE of one-sixth of the MPT one. The lone event which shows no clear difference between the two methods is a 'dot com bubble burst'. It was not a systemic event, but one largely contained to the technology sector.

Exhibit 7 – Comparison of decay weighted and tail-implied correlation matrices in stress testing

Shock Factor	Shock Size	Start Period	End Period	RMSE (RiXtrema)	RMSE (MPT)
Diversified Fin	-43.80%	9/3/2008	11/19/2008	0.007	0.034
Italy	-24%	7/6/2011	9/14/2011	0.009	0.058
US Equity	-16%	7/14/1998	10/6/1998	0.016	0.021
Internet	-33.70%	12/29/1999	5/24/2000	0.158	0.179

We have seen that in three major systemic extreme events a RiXtrema model outperformed the traditional method of correlation analysis based on exponential time decay by a wide margin and in a crisis with little to no contagion effects the two measures were roughly in line with each other.

The results of the described analysis, with the conclusions about the fallacies of MPT-based models and effectiveness of RiXtrema solutions, are applicable for asset management tasks at the asset allocation level, as well as individual holdings portfolios.

d. Marginal Distributions

As described in the section above, in the copula approach, the marginal distributions of variables can be separated from the copula. So, $\sigma_j = \sqrt{c_{jj}},\ j = 1, \ldots, m$ the standard deviations of components of X, and

$$D = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \sigma_m \end{pmatrix} \tag{5}$$

Each diagonal element of the matrix (5) represents the standard deviation of the individual factor. This standard deviation is calculated as a square root of the exponentially weighted estimate of variance:

$$\sigma^2{}_j = (1-\lambda) * \sum_{i=1}^{n} \left[\frac{\lambda^{i-1} * r_{j,i}^2}{\sum_{l=1}^{n} (1-\lambda) * \lambda^{l-1}} \right] = \frac{1}{\sum_{l=1}^{n} \lambda^{l-1}} * \sum_{i=1}^{n} \left[\lambda^{i-1} * r_{j,i}^2 \right] \tag{6}$$

λ - exponential decay factor

l, i - indicators of time

$r_{j,i}$ - return of asset j or k at time i

- Note that marginal distributions are not based exclusively or even predominantly on tail events. This is because the goal of the GML model is not to forecast higher variance or to time the market, but to decompose portfolio risks in a realistic manner and to perform robust stress tests and reverse stress tests.

e. Idiosyncratic Risk

Typically, the idiosyncratic risk is arrived at by first calculating the time series residuals of the regression for each security i, see formula (7). The standard deviation of that vector forms the estimate of the idiosyncratic risk or matrix G using the notation introduced in formula (3).

$$\varepsilon_i = R_i - (\beta_{i,1} * F_1 + \beta_{i,2} * F_2 + ... + \beta_N * F_N) \tag{7}$$

RiXtrema models capture joint tail moves through the 'tail copula' described above. Therefore the structure of the residuals must reflect the nature of the extreme periods to properly estimate the contribution of the idiosyncratic risk to the overall portfolio risk. Therefore, we calculate the residual as a three year standard deviation of the result of formula (4). Here it is again:

$$\varepsilon_{xi} = R_i - (\beta_{i,1} * F_1 + \beta_{i,2} * F_2 + ... + \beta_N * F_N + \beta_{i,LQ,PC1} * F_{PC1} ... + \beta_{i,LQ,PC5} * F_{PC5})$$

IV. Summary: How RiXtrema GML and AA Models Address Problems with MPT based models

RiXtrema is a company dedicated to helping financial firms model and manage risks in the tail. The key problems of the traditional paradigm are as follows:

a. Lack of forecasting ability that stems from the fact that risk forecast is a function of realized volatility over the recent period. This problem manifests itself on the asset class level, rather than on the holdings level. RiXtrema's multi asset class Asset Allocation (AA) model is created to look for signs of instability on the asset class level and assess the vulnerability of the portfolio to individual and joint crises on the asset class level. RiXtrema's AA model provides a user with the top down look at tail risks.
b. Inability to properly estimate the diversification benefits available in the tail. As we discussed in sections IIB and IIIC, the traditional methods of estimating correlations pose a real and present danger to the health of any investment firm. Empirically, the tail correlation structure is drastically different from the average structure (where tranquil periods predominate). RiXtrema's proprietary 'tail copula' calibration method ensures that the diversification benefits estimated by the GML model are much more robust and resemble what actually happens in periods of 'global fear'. The 'tail-implied copula' mechanism is meant to capture the changes in the extreme period interrelationships between factors in cases of global instability.
c. Improper estimation of liquidity risk. Traditional models view liquidity as mostly 'market impact' and assume that liquidity risk of a given holding can be measured by looking at some standard ratios (e.g. volume/float for equities or bid/ask for fixed income) and by observing normal periods. In our view this leaves out the most important part of liquidity risk, namely the illiquidity that appears when markets are in crisis. In such periods supposedly neutral (according to the MPT paradigm) portfolios all of a sudden become directional and two holdings with a similar average liquidity profiles may start to behave very differently. The GML model includes a special

proprietary procedure called "RiXtrema Crisis Illiquidity", which analyzes extreme periods in local markets to help users to uncover hidden liquidity bets that will become apparent only in crisis.

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Appendix A: GML Fixed Income Factor Breakdown[12]

Factor Code	Description
USG6MO	6 Months US Government Curve
USG1Y	1 Year US Government Curve
USG2Y	2 Years US Government Curve
USG3Y	3 Years US Government Curve
USG5Y	5 Years US Government Curve
USG7Y	7 Years US Government Curve
USG10Y	10 Years US Government Curve

[12] GML model is customized by adding specific factors requested by the client.

USG20Y	20 Years US Government Curve
USG30Y	30 Years US Government Curve
USS6MO	6 Months USD Swap Curve
USS1Y	1 Year USD Swap Curve
USS2Y	2 Years USD Swap Curve
USS3Y	3 Years USD Swap Curve
USS5Y	5 Years USD Swap Curve
USS7Y	7 Years USD Swap Curve
USS10Y	10 Years USD Swap Curve
USS20Y	20 Years USD Swap Curve
USS30Y	30 Years USD Swap Curve
USM6MO	6 Months US Muni Curve
USM1Y	1 Year US Muni Curve
USM2Y	2 Years US Muni Curve
USM3Y	3 Years US Muni Curve
USM5Y	5 Years US Muni Curve
USM7Y	7 Years US Muni Curve
USM10Y	10 Years US Muni Curve
USM20Y	20 Years US Muni Curve
USM30Y	30 Years US Muni Curve
JPC6MO	6 Months Japan Government Curve
JPC1Y	1 Year Japan Government Curve
JPC2Y	2 Years Japan Government Curve
JPC3Y	3 Years Japan Government Curve
JPC5Y	5 Years Japan Government Curve
JPC7Y	7 Years Japan Government Curve
JPC10Y	10 Years Japan Government Curve
JPC20Y	20 Years Japan Government Curve
JPC30Y	30 Years Japan Government Curve
JPS6MO	6 Months Yen Swap Curve
JPS1Y	1 Year Yen Swap Curve
JPS2Y	2 Years Yen Swap Curve
JPS3Y	3 Years Yen Swap Curve
JPS5Y	5 Years Yen Swap Curve
JPS7Y	7 Years Yen Swap Curve
JPS10Y	10 Years Yen Swap Curve
JPS20Y	20 Years Yen Swap Curve
JPS30Y	30 Years Yen Swap Curve

EUC6MO	6 Months EU Government Curve
EUC1Y	1 Year EU Government Curve
EUC2Y	2 Years EU Government Curve
EUC3Y	3 Years EU Government Curve
EUC5Y	5 Years EU Government Curve
EUC7Y	7 Years EU Government Curve
EUC10Y	10 Years EU Government Curve
EUC20Y	20 Years EU Government Curve
EUC30Y	30 Years EU Government Curve
EUS6MO	6 Months EUR Swap Curve
EUS1Y	1 Year EUR Swap Curve
EUS2Y	2 Years EUR Swap Curve
EUS3Y	3 Years EUR Swap Curve
EUS5Y	5 Years EUR Swap Curve
EUS7Y	7 Years EUR Swap Curve
EUS10Y	10 Years EUR Swap Curve
EUS20Y	20 Years EUR Swap Curve
EUS30Y	30 Years EUR Swap Curve
UKS6MO	6 Months GBP Swap Curve
UKS1Y	1 Year GBP Swap Curve
UKS2Y	2 Years GBP Swap Curve
UKS3Y	3 Years GBP Swap Curve
UKS5Y	5 Years GBP Swap Curve
UKS7Y	7 Years GBP Swap Curve
UKS10Y	10 Years GBP Swap Curve
UKS20Y	20 Years GBP Swap Curve
UKS30Y	30 Years GBP Swap Curve
UKC6MO	6 Months UK Government Curve
UKC1Y	1 Year UK Government Curve
UKC2Y	2 Years UK Government Curve
UKC3Y	3 Years UK Government Curve
UKC5Y	5 Years UK Government Curve
UKC7Y	7 Years UK Government Curve
UKC10Y	10 Years UK Government Curve
UKC20Y	20 Years UK Government Curve
UKC30Y	30 Years UK Government Curve
CAS6MO	6 Months CAD Swap Curve
CAS1Y	1 Year CAD Swap Curve

CAS2Y	2 Years CAD Swap Curve
CAS3Y	3 Years CAD Swap Curve
CAS5Y	5 Years CAD Swap Curve
CAS7Y	7 Years CAD Swap Curve
CAS10Y	10 Years CAD Swap Curve
CAS20Y	20 Years CAD Swap Curve
CAS30Y	30 Years CAD Swap Curve
CAC6MO	6 Months Canada Government Curve
CAC1Y	1 Year Canada Government Curve
CAC2Y	2 Years Canada Government Curve
CAC3Y	3 Years Canada Government Curve
CAC5Y	5 Years Canada Government Curve
CAC7Y	7 Years Canada Government Curve
CAC10Y	10 Years Canada Government Curve
CAC20Y	20 Years Canada Government Curve
CAC30Y	30 Years Canada Government Curve
AUS6MO	6 Months AUD Swap Curve
AUS1Y	1 Year AUD Swap Curve
AUS2Y	2 Years AUD Swap Curve
AUS3Y	3 Years AUD Swap Curve
AUS5Y	5 Years AUD Swap Curve
AUS7Y	7 Years AUD Swap Curve
AUS10Y	10 Years AUD Swap Curve
AUS20Y	20 Years AUD Swap Curve
AUS30Y	30 Years AUD Swap Curve
AUC6MO	6 Months AUD Swap Curve
AUC1Y	1 Year Australia Government Curve
AUC2Y	2 Years Australia Government Curve
AUC3Y	3 Years Australia Government Curve
AUC5Y	5 Years Australia Government Curve
AUC7Y	7 Years Australia Government Curve
AUC10Y	10 Years Australia Government Curve
AUC20Y	20 Years Australia Government Curve
AUC30Y	30 Years Australia Government Curve
CHC6MO	6 Months Switzerland Government Curve
CHC1Y	1 Year Switzerland Government Curve
CHC2Y	2 Years Switzerland Government Curve
CHC3Y	3 Years Switzerland Government Curve

CHC5Y	5 Years Switzerland Government Curve
CHC7Y	7 Years Switzerland Government Curve
CHC10Y	10 Years Switzerland Government Curve
CHC20Y	20 Years Switzerland Government Curve
CHC30Y	30 Years Switzerland Government Curve
CHS6MO	6 Months CHF Swap Curve
CHS1Y	1 Year CHF Swap Curve
CHS2Y	2 Years CHF Swap Curve
CHS3Y	3 Years CHF Swap Curve
CHS5Y	5 Years CHF Swap Curve
CHS7Y	7 Years CHF Swap Curve
CHS10Y	10 Years CHF Swap Curve
CHS20Y	20 Years CHF Swap Curve
CHS30Y	30 Years CHF Swap Curve
Industrial AAA	Industrial AAA Spread
Industrial AA	Industrial AA Spread
Industrial A	Industrial A Spread
Industrial BBB	Industrial BBB Spread
Financial AAA	Financial AAA Spread
Financial AA	Financial AA Spread
Financial A	Financial A Spread
Financial BBB	Financial BBB Spread
Utilities AA	Utilities AA Spread
Utilities A	Utilities A Spread
Utilities BBB	Utilities BBB Spread
HYBB	HYBB Spread
HYB	HYB Spread
HYCCC	HYCCC Spread
HYBB_B	HYBB_B Spread
Sovereign Agency	Sovereign Agency Spread
Mtge	Mtge Spread
Muni AAA	Muni AAA Spread
Muni AA	Muni AA Spread
Muni A	Muni A Spread
Muni BBB	Muni BBB Spread
Credit Card ABS	Credit Card ABS Spread
Home Equity ABS	Home Equity ABS Spread
Automobile ABS	Automobile ABS Spread

Corporate AAA	Corporate AAA Spread
Corporate AA	Corporate AA Spread
Corporate A	Corporate A Spread
Corporate BBB	Corporate BBB Spread
MXC6MO	6 Months Mexico Government Curve
MXC1Y	1 Year Mexico Government Curve
MXC2Y	2 Years Mexico Government Curve
MXC3Y	3 Years Mexico Government Curve
MXC5Y	5 Years Mexico Government Curve
MXC7Y	7 Years Mexico Government Curve
MXC10Y	10 Years Mexico Government Curve
MXC20Y	20 Years Mexico Government Curve
MXC30Y	30 Years Mexico Government Curve
MXS6MO	6 Months MXN Swap Curve
MXS1Y	1 Year MXN Swap Curve
MXS2Y	2 Years MXN Swap Curve
MXS3Y	3 Years MXN Swap Curve
MXS5Y	5 Years MXN Swap Curve
MXS7Y	7 Years MXN Swap Curve
MXS10Y	10 Years MXN Swap Curve
MXS20Y	20 Years MXN Swap Curve
MXS30Y	30 Years MXN Swap Curve
MXCORP	Mexico Sovereign Spread
CANCORP	Canada Corporate Spread
LXCORP	Luxemburg Corporate Spread
FRCORP	France Corporate Spread
UKCORP	UK Corporate Spread
EUFIN	
ZAC6MO	6 Months South Africa Government Curve
ZAC1Y	1 Year South Africa Government Curve
ZAC2Y	2 Years South Africa Government Curve
ZAC3Y	3 Years South Africa Government Curve
ZAC5Y	5 Years South Africa Government Curve
ZAC7Y	7 Years South Africa Government Curve
ZAC10Y	10 Years South Africa Government Curve
ZAC20Y	20 Years South Africa Government Curve
ZAC30Y	30 Years South Africa Government Curve
MYC6MO	6 Months Malaysia Government Curve

MYC1Y	1 Year Malaysia Government Curve
MYC2Y	2 Years Malaysia Government Curve
MYC3Y	3 Years Malaysia Government Curve
MYC5Y	5 Years Malaysia Government Curve
MYC7Y	7 Years Malaysia Government Curve
MYC10Y	10 Years Malaysia Government Curve
MYC20Y	20 Years Malaysia Government Curve
MYC30Y	30 Years Malaysia Government Curve
PLC6MO	6 Months Poland Government Curve
PLC1Y	1 Year Poland Government Curve
PLC2Y	2 Years Poland Government Curve
PLC3Y	3 Years Poland Government Curve
PLC5Y	5 Years Poland Government Curve
PLC7Y	7 Years Poland Government Curve
PLC10Y	10 Years Poland Government Curve
PLC20Y	20 Years Poland Government Curve
PLC30Y	30 Years Poland Government Curve
TRC6MO	6 Months Turkey Government Curve
TRC1Y	1 Year Turkey Government Curve
TRC2Y	2 Years Turkey Government Curve
TRC3Y	3 Years Turkey Government Curve
TRC5Y	5 Years Turkey Government Curve
TRC7Y	7 Years Turkey Government Curve
TRC10Y	10 Years Turkey Government Curve
TRC20Y	20 Years Turkey Government Curve
TRC30Y	30 Years Turkey Government Curve
SOV_IR	Ireland Sovereign Spread
SOV_BE	Belgian Sovereign Spread
SOV_SP	Spain Sovereign Spread
SOV_FR	France Sovereign Spread
SOV_EN	UK Sovereign Spread
SOV_PO	Poland Sovereign Spread
SOV_IT	Italy Sovereign Spread